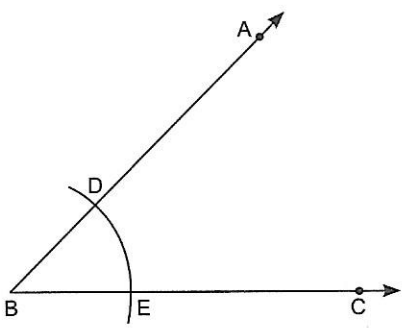
CONSTRUCTIONS

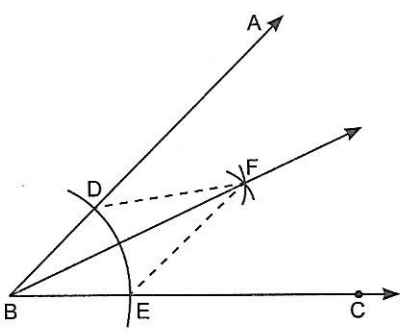
Introduction

- **Geometrical Construction:** It is the process of drawing a geometrical figure using only two instruments – a ruler and a compass. Although, protractor may be used for drawing non-standard angles.
It is an important part of the study of geometry for understanding the concepts that we have learnt in theoretical geometry.

Basic Constructions

- **To construct the bisector of a given angle.**
Given: An angle ABC.
Required: To construct the bisector of ∠ABC.
Steps of construction:
(*i*) Draw any acute angle ABC. With B as centre and of any radius, draw an arc DE to intersect BA at D and BC at E.
(*ii*) With E as centre and any suitable radius ($> \frac{1}{2}$DE), draw an arc. Also, with D as centre, draw another arc of the same radius to intersect the previous arc at F.
(*iii*) Draw the ray BF as shown in the figure. This ray BF is the required bisector of ∠ABC.





Verification by theoretical concept:
Join DF and EF.
In ΔBEF and ΔBDF,

	BE = BD	(Radius of the same arc)
	EF = DF	(Arcs of equal radii)
	BF = BF	(Common)
∴	ΔBEF ≅ ΔBDF	(SSS congruence rule)
⇒	∠EBF = ∠DBF	(CPCT)

∴ BF is the angle bisector of ∠ABC. Hence, justified the construction of angle bisector.

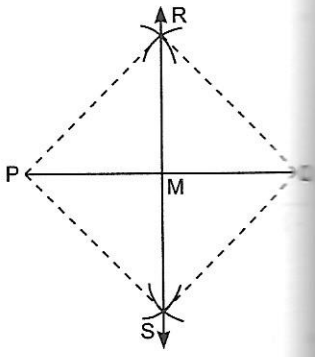
- **To construct the perpendicular bisector of a given line segment.**
Given: A line segment PQ.
Required: To construct a perpendicular bisector of the line segment PQ.

Steps of construction:

(*i*) Taking P and Q as centres and radius more than $\frac{1}{2}$PQ, draw arcs, above and below of the line segment PQ.

(*ii*) Let these arcs above the line segment PQ intersect at R and below the line segment PQ at S. Join RS, which intersects PQ at M.

(*iii*) Line RMS is the required perpendicular bisector of PQ.



Justification of construction:

Join PR, QR, PS and QS

In ΔRPS and ΔRQS,

$PR = QR$ (Arcs of equal radi

$PS = QS$ (Arcs of equal radi

$PQ = PQ$ (Commo

$\therefore \quad \Delta RPS \cong \Delta RQS$ (SSS congruence rul

$\Rightarrow \quad \angle PRM = \angle QRM$ (CPC

Now, in ΔRMP and ΔRMQ,

$PR = QR$ (Arcs of equal radi

$RM = RM$ (Commo

$\angle PRM = \angle QRM$ (Proved abo

$\therefore \quad \Delta RMP \cong \Delta RMQ$ (SSS congruence rul

$\Rightarrow \quad \angle RMP = \angle RMQ$ (CPC

and $PM = QM$ (CPC

But $\angle RMP + \angle RMQ = 180°$ (Linear pair axio

$\Rightarrow \quad 2\angle RMP = 180°$

$\Rightarrow \quad \angle RMP = 90°$

So $\angle RMQ = 90°$

Therefore, RM, i.e. RMS is the perpendicular bisector of PQ. Hence, justified the construction.

- **To construct an angle of 60° at the initial point of a given ray.**

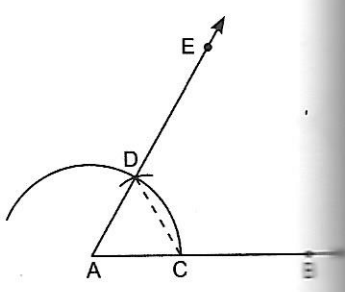
Steps of construction:

(*i*) Take a ray AB with initial point A.

(*ii*) Taking A as centre and with a suitable radius, draw an arc which intersects AB, say at a point C.

(*iii*) Taking C as centre and with the same radius as in step (*ii*), draw an arc, intersecting the previously drawn arc, say at a point D.

(*iv*) Draw the ray AE passing through D. Then, $\angle EAB$ is the required angle of 60°.



Justification:

Join CD.

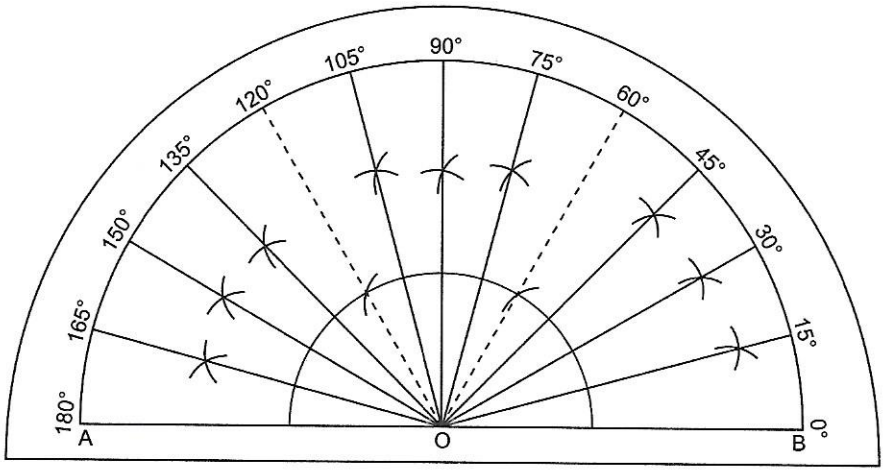
Then, $AC = CD = DA$ (By construct

Therefore, ΔACD is an equilateral triangle and we know that each angle of an equilateral triangle is

$\therefore \quad \angle DAC = 60°$ or $\angle EAB = 60°$

Hence, our construction for an angle of 60° is justified.

- **Angle bisectors:** The following figure shows the constructions of various angles which are multiples of 15° in the anti-clockwise direction drawn with the help of a ruler and a compass.



Construction of a Triangle

- A triangle is unique if:
  (*i*) two sides and the included angle is given,
  (*ii*) three sides are given,
  (*iii*) two angles and included side is given and,
  (*iv*) in a right triangle, hypotenuse and one side is given.

If above combination(s) are not given, then we have to find the way to construct the triangle uniquely. For example: To construct a triangle,

(*i*) given its base, a base angle and
  (*a*) sum of other two sides.
  (*b*) the difference of the other two sides.
(*ii*) given its perimeter and its two base angles.

We will see how to construct such triangles in the next exercise.

Note: The construction of a triangle is not possible, if the sum of any two sides of a triangle is less than or equal to the third side, i.e. AB + AC ≤ BC.

SOLVED QUESTIONS BASED ON EXERCISES 11.1 AND 11.2

Very Short Answer Type Questions [1 Mark]

1. Is it possible to construct the angle of 37.5° with the help of ruler and compass?

Sol. Yes, it is possible because by constructing 75° angle and bisecting it, we can obtain 37.5° angle.

2. Do you agree with the statement, 'ΔXYZ can be constructed, if $\angle Y = 90°$, $\angle Z = 75°$ and XY + YZ + ZX = 11.5 cm'. **[CBSE 2016]**

Sol. Yes, because two base angles and perimeter is given and $\angle Y + \angle Z = 90° + 75° = 165° < 180°$

3. Can you construct a ΔABC, if AB = 6.5 cm, $\angle A = 60°$ and BC + AC = 11 cm. **[CBSE 2016]**

Sol. Yes, with the given dimensions, we can construct the ΔABC because BC + AC > AB.

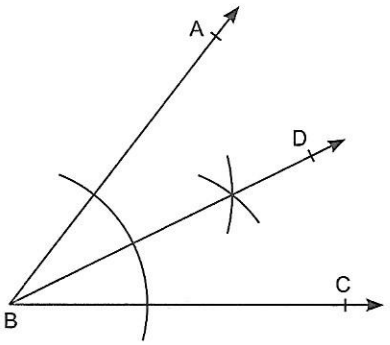
4. In how many parts, bisector of an angle divides it? **[CBSE 2015]**

Sol. The bisector of an angle divides the angle into two equal parts.

Short Answer Type Questions I [2 Marks]

5. Using protractor, draw an angle of 52°. Can you divide this angle into two equal parts. Show it. [CBSE 2014]

Sol. Yes, we can divide $\angle ABC = 52°$ into two equal parts by bisecting it as shown in the figure.



6. Construct a triangle whose sides are in the ratio 1 : 3 : 5 and whose perimeter is 18 cm. [CBSE 2016]

Sol. Given ratio of sides of a triangle = 1 : 3 : 5

Let the length of sides of a triangle be x, $3x$ and $5x$ respectively

Perimeter of triangle = 18 cm

$\Rightarrow \quad x + 3x + 5x = 18$

$\Rightarrow \quad 9x = 18$

$\Rightarrow \quad x = 2$ cm

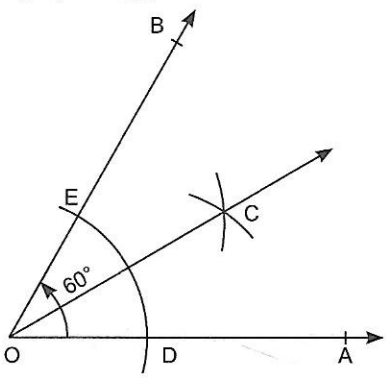
$\therefore$ Sides of ΔABC are AB = 2 cm, BC = 6 cm and AC = 10 cm.

Here, we find that AB + BC < AC.

So, construction of given triangle would not be possible.

7. Draw an angle of an equilateral triangle, using protractor. Bisect it using compass. [CBSE 2016]

Sol.



Each angle of an equilateral triangle is 60°

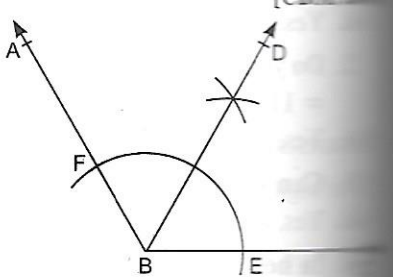
$\therefore$ According to question, $\angle AOB = 60°$.

$\Rightarrow$ OC is the bisector of $\angle AOB$.

8. Draw any obtuse angle. Bisect it using compass. [CBSE 2016]

Sol. Let $\angle ABC = 120°$

Draw the bisector BD of $\angle ABC$ as shown in the figure.



9. Is it possible to construct a triangle of given sides as 44 mm, 9.5 cm and 46 mm? Justify your answer.

Sol. Let $AB = 44 \text{ mm} = 4.4 \text{ cm}$

$BC = 9.5 \text{ cm}$

$AC = 46 \text{ mm} = 4.6 \text{ cm}$

Here, $AB + AC = 4.4 \text{ cm} + 4.6 \text{ cm} = 9 \text{ cm}$

$\Rightarrow$ $AB + AC < BC$

$\therefore$ No such triangle would be constructed because sum of two sides of a triangle is never less than the third side.

10. Construct an equilateral triangle, given its one side is 5 cm. **[CBSE 2012]**

Sol. We know that all sides of an equilateral triangle are equal

$\therefore$ In ΔABC, $AB = BC = CA = 5$ cm.

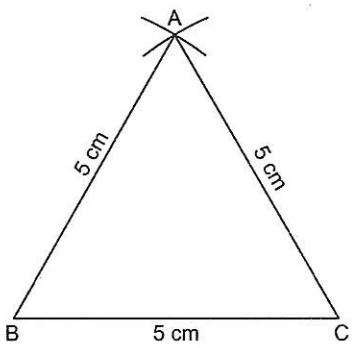
Steps of construction:

(*i*) Draw a line segment, BC = 5 cm

(*ii*) Taking B and C as centres and radius equal to 5 cm, draw arcs which intersect each other at A.

(*iii*) Join AB and AC.

Thus, ΔABC is the required equilateral triangle.



Short Answer Type Questions II [3 Marks]

11. Construct a triangle ABC in which BC = 5 cm, $\angle B = 75°$ and AB + AC = 9 cm. **[CBSE 2012]**

Sol. Steps of construction:

(*i*) Draw a line segment BC = 5 cm. At point B, construct an $\angle XBC = 75°$

(*ii*) Cut a line segment BD = AB + AC = 9 cm from the ray BX.

(*iii*) Join CD.

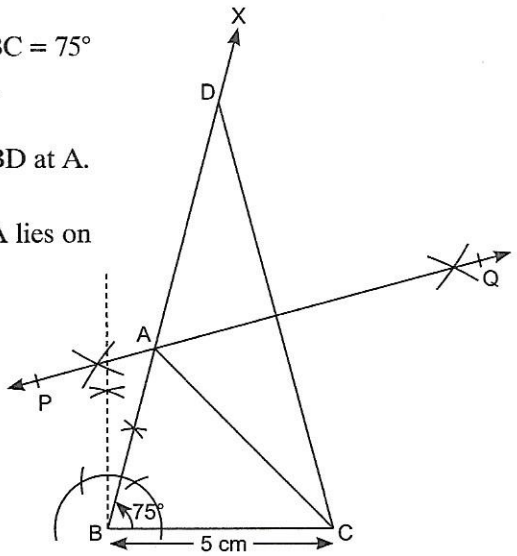
(*iv*) Draw the perpendicular bisector PQ of CD which intersects BD at A.

(*v*) Join AC.

(*vi*) Then, ΔABC is the required triangle. This is because point A lies on the perpendicular bisector of CD

$\therefore$ $AD = AC$

$\Rightarrow$ $BD = AB + AD = AB + AC$



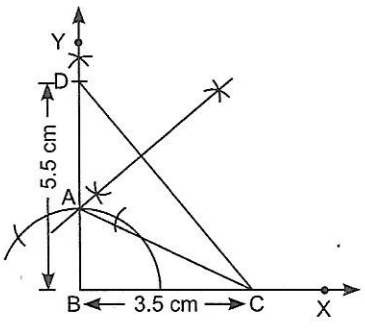
12. Construct a right triangle in which one side is 3.5 cm and sum of the other side and hypotenuse is 5.5 cm.

Sol. We are given one side = 3.5 cm and sum of other side and hypotenuse = 5.5 cm.

Steps of Constructon:

1. Draw a ray BX and cut off a line segment BC = 3.5 cm from it.
2. Construct $\angle XBY = 90°$.
3. From BY, cut off a line segment BD = 5.5 cm.
4. Join CD.
5. Draw the perpendicular bisector of CD intersecting BD at a point A.
6. Join AC.

So, ΔABC is the required triangle.



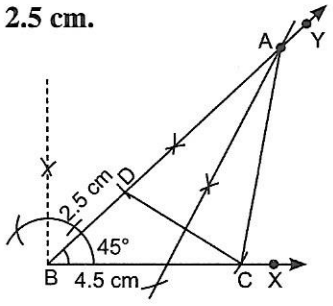
13. Construct a triangle ABC in which BC = 4.5 cm, $\angle B = 45°$ and AB – AC = 2.5 cm.

Sol. We are given BC = 4.5 cm, $\angle B = 45°$ and AB – AC = 2.5 cm.

Steps of Construction:

1. Draw a ray BX and cut off a line segment BC = 4.5 cm from it.
2. Construct $\angle XBY = 45°$.
3. Cut off a line segment BD = 2.5 cm from BY.
4. Join CD.
5. Draw the perpendicular bisector of CD cutting BY at a point A.
6. Join AC.

So, ΔABC is the required triangle.



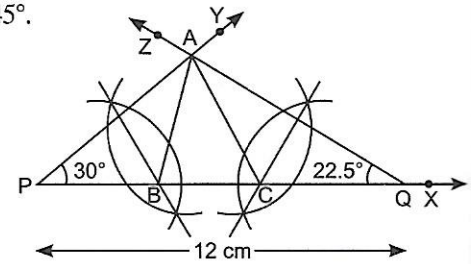
14. Construct a triangle ABC whose perimeter is 12 cm, $\angle B = 60°$ and $\angle C = 45°$.

Sol. We are given that perimeter of triangle = 12 cm, $\angle B = 60°$ and $\angle C = 45°$.

Steps of Construction:

1. Draw a ray PX and cut off a line segment PQ = 12 cm from it.
2. At P, construct $\angle YPQ = 30° \left(= \frac{1}{2} \times 60°\right)$.
3. At Q, construct $\angle ZQP = 22.5° \left(= \frac{1}{2} \times 45°\right)$.
4. Let the rays PY and QZ intersect at A.
5. Draw the perpendicular bisector of AP intersecting PQ at a point B.
6. Draw the perpendicular bisector of AQ intersecting PQ at a point C.
7. Join AB and AC.

So, ΔABC is the required triangle.



15. Construct a triangle ABC in which $\angle B = 60°$, $\angle C = 75°$ and perpendicular from the vertex A to the base BC is 5 cm.

Sol. In ΔABC, $\angle A + \angle B + \angle C = 180°$ (Angle sum property of a triangle)

$\Rightarrow$ $\angle A + 60° + 75° = 180° \Rightarrow \angle A = 180° - 135° = 45°$

Steps of construction:

(*i*) Draw a line BX.

(*ii*) At point B, construct $\angle B = 60°$, i.e. $\angle XBY = 60°$

(*iii*) Draw two arcs R and S with radius equal to 5 cm from point B and from any other point D on BX as shown.

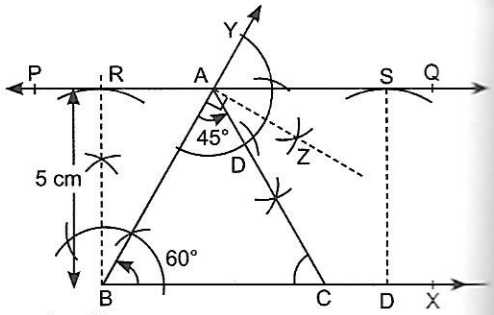
(*iv*) Draw a ray PQ touches the R and S in such a way that, RS || BX and distance between them is BR = DS = 5 cm.

(*v*) Let BY intersect PQ at A.

(*vi*) At point A, construct $\angle ZAB = 90°$.

(*vii*) Bisect $\angle ZAB$ to get $\angle BAC = 45°$. Bisector line intersects BX at point C.

(*viii*) Join AC, then ΔABC is the required triangle.



16. Draw a line segment PQ = 8.4 cm. Divide it into four equal parts using a ruler and a compass.

[CBSE 2014, 2015; HOTS]

Sol. Steps of construction:

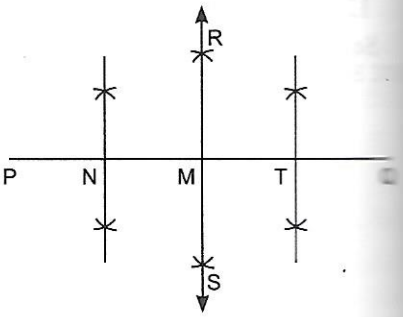
(*i*) Draw a line segment PQ = 8.4 cm.

(*ii*) Taking P and Q as centres and radius more than $\frac{1}{2}$ PQ, draw arcs above and below the line segment PQ intersecting at R and S respectively as shown.

(*iii*) Join RS. Let it intersect PQ at M. The ray RS divides the line segment PQ into two equal parts PM and QM.

(*iv*) In a similar way, draw perpendicular bisectors of PM and QM which divides each PM and QM into two equal parts again as shown.

So, the four equal parts of line segment PQ are PN = NM = MT = TQ. On measuring them, they all are equal to 2.1 cm.



Long Answer Type Questions [4 Marks]

17. Construct a triangle ABC in which BC = 5.8 cm, ∠B = 45° and ∠C = 60°. Construct angle bisectors of ∠B and ∠C and intersect them at point O. Measure ∠BOC. **[CBSE 2016]**

Sol. Steps of construction:

(*i*) Draw a line segment BC = 5.8 cm

(*ii*) At B and C, draw ∠XBC = 45° and ∠YCB = 60°

(*iii*) The rays XB and YC intersect at A. Therefore, ΔABC is the required triangle.

(*iv*) Taking B as centre, and with some radius, draw arcs intersecting XB and BC at E and D respectively.

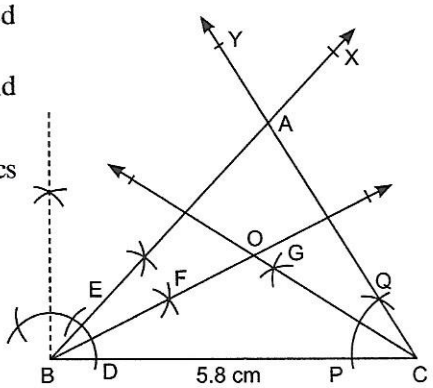
(*v*) Taking D and E as centres with radius greater than $\frac{1}{2}$ DE, draw arcs intersecting each other at F.

(*vi*) Draw the ray BF. It is the angle bisector of ∠B.

(*vii*) Similarly, construct angle bisector CG of ∠C.

(*viii*) Let BF and CG intersect each other at O.

(*ix*) On measuring ∠BOC, we get BOC = 127°



18. Construct a triangle PQR in which ∠R = 45° ∠Q = 60° and PQ + QR + RP = 11 cm.

Sol. Steps of construction:

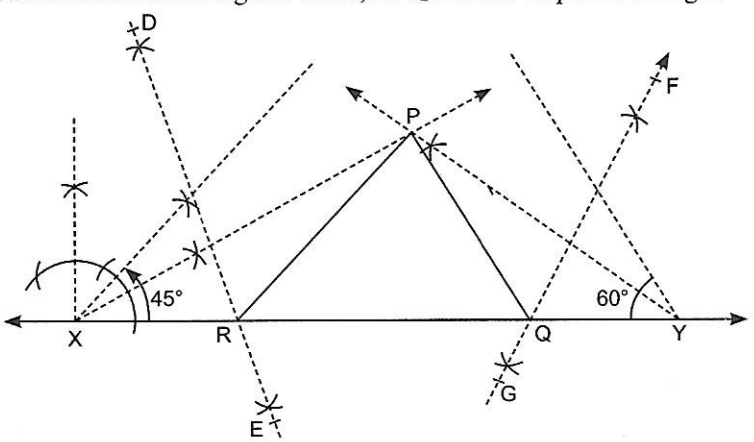
(*i*) Draw a line segment XY = PQ + QR + RP = 11 cm.

(*ii*) At X, construct an angle of 45° and at Y, construct an angle of 60°.

(*iii*) Bisect these angles. Let the bisectors of ∠X and ∠Y intersect each other at a point P.

(*iv*) Draw perpendicular bisector DE of PX to intersect XY at R. Now, draw perpendicular bisector FG of PY to intersect XY at Q.

(*v*) Join PQ and PR as shown in the figure. Then, ΔPQR is the required triangle.



19. Construct a triangle ABC in which BC = 8 cm, ∠B = 30° and AB – AC = 3.5 cm.

Sol. Here, AB > AC, i.e. AB – AC is given.

Steps of construction:

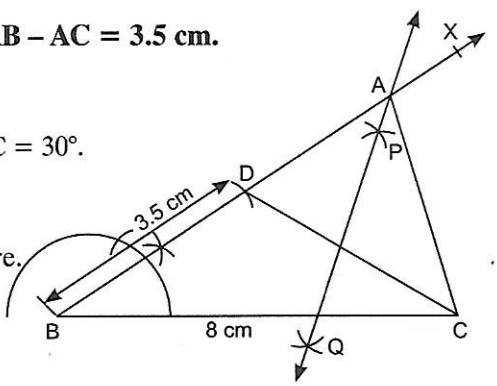
(*i*) Draw the base BC = 8 cm and at point B, make an angle ∠XBC = 30°.

(*ii*) Cut a line segment BD = AB – AC = 3.5 cm from the ray BX.

(*iii*) Join DC and draw the perpendicular bisector PQ of DC.

(*iv*) Let PQ intersect BX at a point A. Join AC as shown in the figure.

Then, ΔABC is the required triangle.



20. Draw any acute angle. Divide it into four equal parts using a ruler and a compass. Measure them using protractor. [CBSE 2014]

Sol. Steps of construction:

(*i*) Draw an acute angle, $\angle ABC = 60°$ (say).

(*ii*) Bisect $\angle ABC$. Join BD. Then $\angle ABD = \angle CBD = \frac{1}{2}\angle ABC = \frac{1}{2} \times 60° = 30°$.

(*iii*) Again bisect $\angle ABD$, join BF as shown. Then

$\angle ABF = \angle FBD = \frac{1}{2}\angle ABD = \frac{1}{2} \times 30° = 15°$.

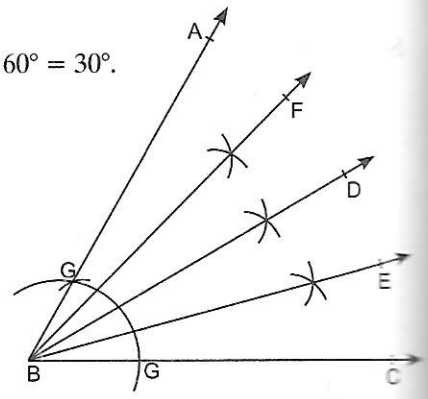
(*iv*) Again bisect $\angle CBD$. Join BE. Then

$\angle DBE = \angle EBC = \frac{1}{2}\angle CBD = \frac{1}{2} \times 30° = 15°$.

Thus, $\angle ABC$ has been divided into four equal parts

$\therefore \quad \angle ABF = \angle FBD = \angle DBE = \angle EBC = \frac{1}{4}\angle ABC = \frac{1}{4} \times 60° = 15°$

On measuring them, we also got each angle equals to 15°.



PRACTICE QUESTIONS BASED ON EXERCISES 11.1 AND 11.2

1. For what value of (AB – AC), the construction of a ΔABC is possible, if BC = 3 cm and $\angle C = 30°$?
[NCERT Exemplar]
2. Find BC – AC, when it is not possible to construct a triangle ABC in which AB = 9 cm and $\angle A = 60°$.
3. Can you construct an angle of measure 44° with the help of a ruler and a compass?
4. Using $\angle MOX = 135°$, construct an angle of 67.5° with the help of a compass and a ruler.
[CBSE 2016]
5. Construct a triangle ABC in which three sides are given of length 6 cm, 4.8 cm and 5 cm respectively.
6. Why the construction of triangle with one right angle and other obtuse angle is not possible?
[CBSE 2016]
7. Construct a right triangle PQR with $\angle Q = 90°$ and whose length of its longest side is 8 cm and base is 5.5 cm.
8. Construct a triangle XYZ in which XY = 6 cm, YZ =4 cm and median ZM = 3.5 cm.
9. Construct an angle of $52\frac{1°}{2}$ using a ruler and a compass.
10. Construct an angle of 150°. Bisect it using a ruler and a compass.
11. Construct an equilateral ΔPQR, given its one side is 8 cm.
12. Construct a triangle ABC in which BC = 7.5 cm, $\angle B = 45°$ and AB – AC = 2.5 cm.
13. Construct an equilateral triangle, if its altitude is 3.6 cm. Give justification for your construction.
14. Construct an isosceles ΔPQR in which base QR = 5.8 cm and altitude from P on QR is 4 cm. Measure the equal sides.
15. Construct a triangle ABC, such that BC = 5 cm, $\angle B = 20°$ and difference of sides AB and AC is 2.5 cm.
16. Construct a triangle ABC in which AB = 6.4 cm, sum of the other two sides is 8 cm and $\angle B = 60°$.
[HOTS]
17. Construct a triangle PQR, such that sum of its all sides is 10.4 cm, $\angle B = 60°$ and $\angle C = 40°$.
18. Draw a line segment of 10 cm. Divide it into four equal parts using a compass.
19. Construct $\angle CAB = 75°$ using a compass. Now using $\angle CAB$, construct $\frac{1}{2}\angle CAB$ and $2\angle CAB$ with the help of a compass. [CBSE 2014]
20. Construct a triangle in which $\angle B = 120°$, BC = 3 cm and KB + KC = 4.7 cm. Give only justification.

Value Based Questions

1. A non-government organisation adopted a triangular plot of sides 11 m, 15 m and 21 m to construct a building to save poor people during winter season.
(*i*) Construct the boundaries for the above plot by taking the scale in cm (instead of m).
(*ii*) What ideas are being promoted by this organisation?

2. A group of students under their colleges initiative "We Care" approached Lions Club. By understanding their philanthropic initiative, the club agreed to construct a child care centre on one of their right triangular shaped plot having one side 13.5 m and sum of other side and hypotenuse would be 15.5 m.
   (*i*) Construct the triangle taking measurements of sides in proportion similar to that of plot.
   (*ii*) What ideas are promoted by the group of students?

3. As a part of Corporate Social Responsibility (CSR) activity, an industrialist wishes to construct a hospital for animals on a triangular shaped plot.
   (*i*) Construct a triangle for the same in which BC = 40.5 m, $\angle B = 45°$ and AB – AC = 20.5 m by using proper scale.
   (*ii*) What ideas are promoted by the industrialist?

INTEGRATED EXERCISE

Very Short Answer Type Questions [1 Mark]

1. State the condition for which construction of triangle is not possible.
2. Can a triangle ABC be constructed for the measurements of BC = 6 cm, $\angle C = 40°$ and AC – AB = 5 cm.
3. In ΔABC, $\angle B = 110°$, $\angle C = 90°$ and AB + BC + CA = 12 cm. Can this triangle be constructed?
4. In ΔPQR, $\angle P = 75°$, $\angle Q = 30°$ and PQ + QR + RP = 8.4 cm. Can this triangle be constructed?
5. With the help of a ruler and a compass, can you construct an angle of $7\frac{1}{2}^\circ$?

Short Answer Type Questions I [2 Marks]

6. Draw an angle of 110° with the help of a protractor and bisect it. Measure each angle.
7. Draw a straight angle. Using a compass, bisect it. Name the angles obtained.
8. (*i*) The construction of an angle of $22\frac{1}{2}^\circ$ is possible with the help of a ruler and a compass. Give reasons.
   (*ii*) Is it possible to construct a triangle of given sides as 6.4 cm, 106 mm and 0.036 m? Justify your answer.
9. Draw a line segment AB = 8 cm. By using a ruler and a compass, obtain a line segment of length $\left(\frac{3}{4}\right)$ AB.
10. Draw an equilateral triangle whose altitude is 6 cm.

Short Answer Type Questions II [3 Marks]

11. Draw a linear pair of angles. Bisect each of the two angles. Verify that two bisecting rays are perpendicular to each other.
12. Draw a line segment AB of 4 cm in length. Draw a perpendicular line through A and B respectively. Are these lines parallel? **[NCERT Exemplar; HOTS]**
13. Draw an angle of 80° with the help of protractor. Then, construct angles of 40°, 160° and 120°. **[NCERT Exemplar]**
14. Construct a triangle ABC in which BC = 5 cm, $\angle B = 60°$ and AC + AB = 7.5 cm. **[NCERT Exemplar]**
15. Draw lines AB and CD intersecting each other at point O. Measure a pair of vertically opposite angles. Bisect them. Are the bisecting rays forming a straight line? **[CBSE 2016]**
16. Draw lines l and m intersected by a transversal t. Construct angle bisectors of the interior angles on the same side of the transversal. **[HOTS]**

Long Answer Type Questions [4 Marks]

17. Construct a triangle PQR, given that QR = 6 cm, $\angle Q = 45°$ and QP – PR = 2.5 cm and justify the construction.
18. A right triangle is given when one side is 4 cm and sum of other side and hypotenuse is 8 cm. Construct this triangle and give its justification. **[NCERT Exemplar]**
19. Construct an equilateral triangle, if its altitude is 3.2 cm. **[NCERT Exemplar]**
20. Construct a triangle, if its perimeter is 10.4 cm and two angles are 45° and 120°. Give justification of the construction. **[NCERT Exemplar]**

ASSESS YOURSELF

1. Can an angle of $42\frac{1}{2}°$ be constructed with the help of ruler and compass? Give reason.
2. Is it possible to construct ΔABC in which $\angle B = 30°$, $\angle C = 105°$ and $AB + BC + CA = 12$ cm? Justify your answer.
3. Why we cannot construct a triangle PQR if $\angle P = 60°$, $PQ = 6$ cm, $PR + QR = 5$ cm but construction of triangle PQR is possible when $\angle P = 60°$, $PQ = 6$ cm and $PR - QR = 5$ cm.
4. Draw a line segment of length 5.8 cm. Bisect it and measure the length of each part.
5. Construct an equilateral triangle each of whose sides measures 4.2 cm.
6. Draw a right-angled triangle whose hypotenuse measures 6 cm and the length of one of whose sides containing the right angle is 4 cm.
7. Construct an angle of $37\frac{1}{2}°$.
8. Construct a triangle whose sides are 3.6 cm, 3 cm and 4.8 cm. Bisect the smallest angle and measure each part. **[NCERT Exemplar]**
9. Construct a ΔABC in which $\angle B = 60°$, $\angle C = 45°$ and $BC = 5.5$ cm.
10. Construct a ΔABC in which $AB + AC = 7.2$ cm, $BC = 3.0$ cm and $\angle B = 45°$.
11. Construct a ΔPQR in which three sides are in the ratio 2 : 3 : 3 and perimeter of triangle is 24 cm. **[CBSE 2016]**
12. Construct a triangle having sides of 6.2 cm, 7.3 cm and 6 cm. Measure all the three angles. Bisect the smallest and the largest angle. Measure any acute angle formed by the bisecting rays at the point of intersection. **[CBSE 2014]**
13. Construct ΔABC in which $AB = 6.7$ cm, $\angle A = 55°$ and $AC - BC = 1.2$ cm.
14. Construct an isosceles triangle in which sides are x, x and $x - 2$ and perimeter is equal to 13 cm. **[CBSE 2013]**
15. Construct a triangle ABC such that $\angle B = 60°$, $\angle C = 45°$ and $AB + BC + CA = 10$ cm.